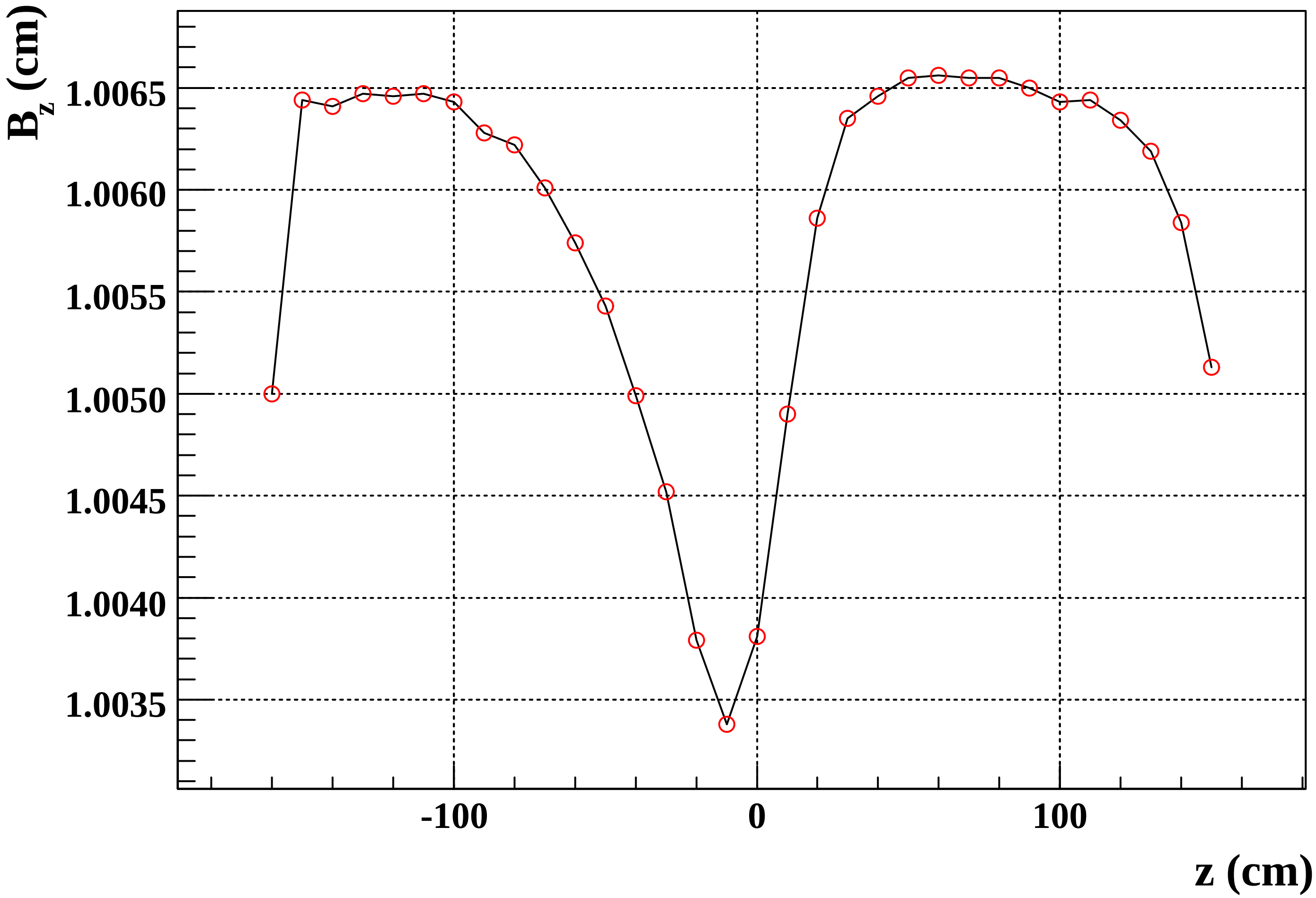
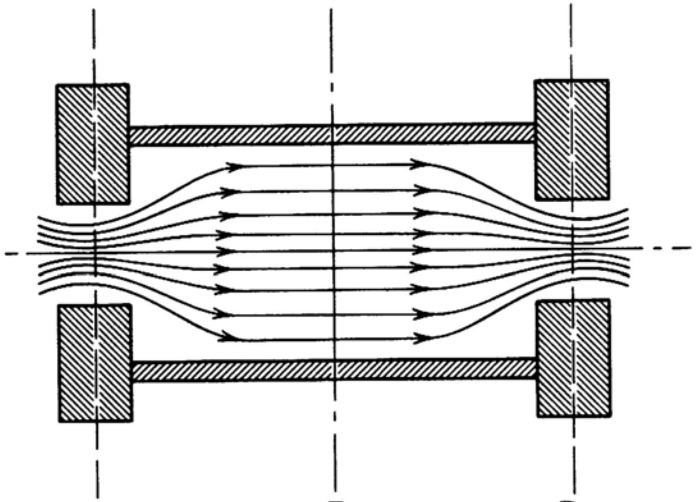
Studies of the Effects of SCS Field Dip



Measured field profile in Z direction (Nov. 07). No shim coils.

Part I: studies with simple field profile

Trapping of Electrons in an Adiabatically Varying B Field



Trapping condition from Jackson:

$$\left|\frac{v_{//0}}{v_{\perp 0}}\right| = \tan^{-1}(\theta) < \sqrt{\frac{B_0}{B_{dip}} - 1}$$

So for B_{dip} = 1.0035 T, B_0 = 1.0065 T, $\theta_c = 86.87^o$

If electrons originated from the center of the trap isotropically, the fraction of trapped electrons is

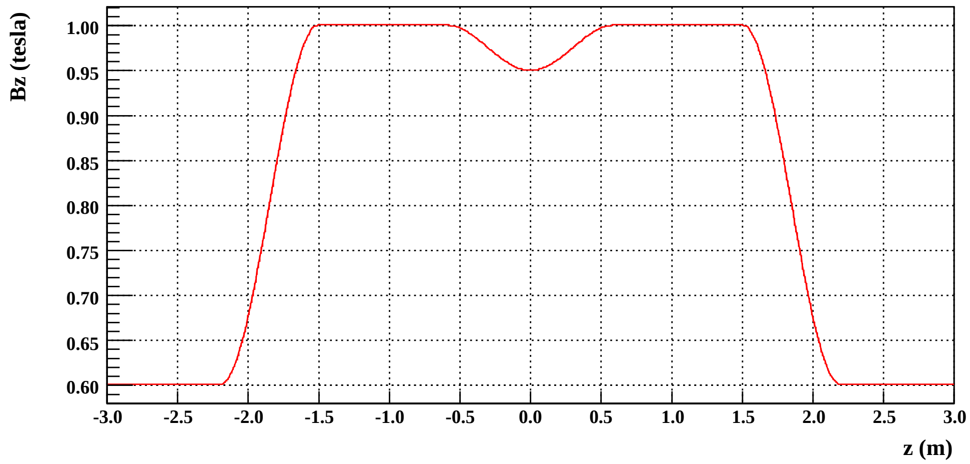
$$f_{trapped} = \frac{2\int_{\theta_c}^{\pi/2} \sin(\theta)d\theta \times 2\pi}{4\pi} = \cos\theta_c$$

For $\theta_c = 86.87^o$,
$f_{trapped} = 5.5\%$

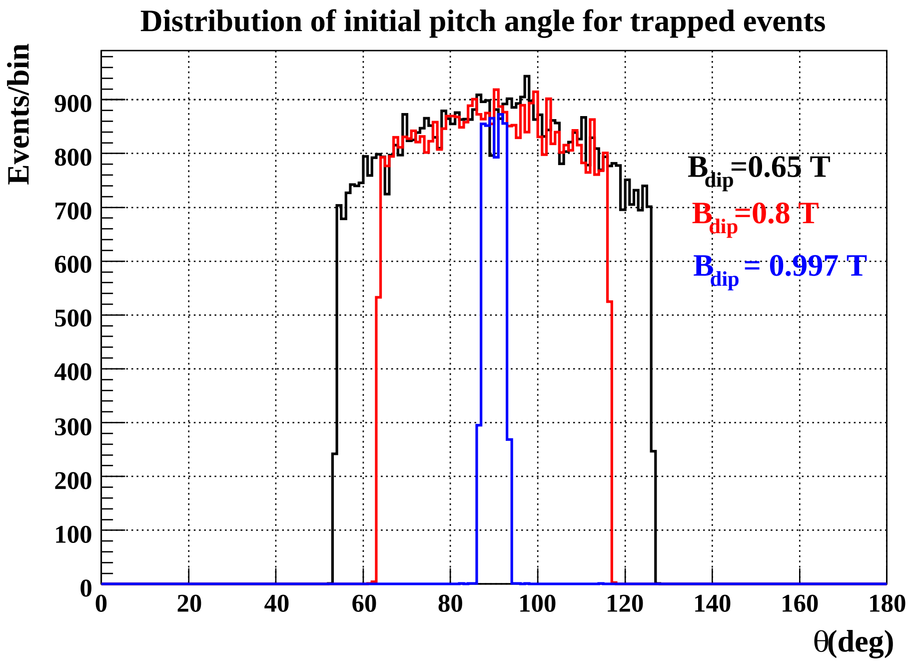
GEANT4 Benchmark

- To cross check, use a field profile with varying Bz in the dip. Generate electrons from the center of the dip uniformly.
- The interpolation between Bz points is made with cos(z) functions. (Bx, By, Bz) satisfy $\nabla \cdot B = 0$
- Generated neutron beta electrons isotropically
- Use 4th order Rutta-Kunge integrator
- Limit tracking step size to 100 μm

Dip between ±0.6 m



Benchmark Results



Angular distribution make sense.

B_{dip} (T)	$f_{trapped}$ (%)	Expected (%)
0.65	59.4(0.2)	59.2
0.7	54.9(0.2)	54.8
0.8	44.7(0.2)	44.7
0.9	31.6(0.2)	31.6
0.95	22.6(0.2)	22.4
0.997	5.7(0.1)	5.5

Simulation of Semi-real Experiment

Use "idealized" field profile on page 4. Generating neutron betas in the entire fiducial between ± 1.5 m

Naïve Expectation

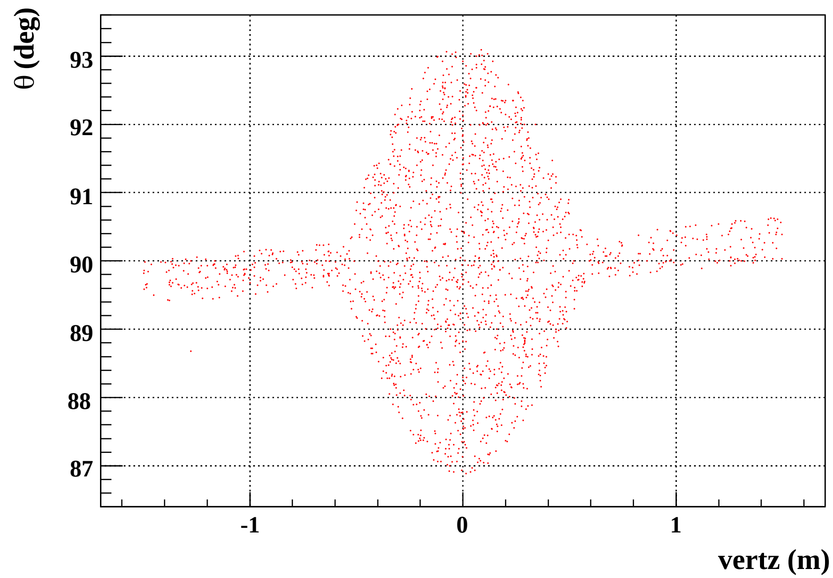
- Electrons originated outside the dip and head toward the dip will travel through it, since the pitch angle becomes more forward in the dip region
- Fraction of the electrons originated inside the trap, with a pitch angle $> \theta_c$, will be trapped. Eventually they will get out due to scattering processes with residual gas molecules

From the Monte Carlo, out of 100 K electron events,

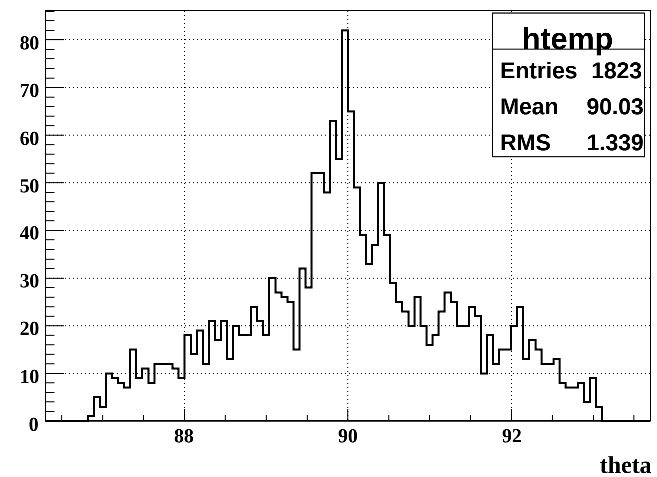
- 1823 (~ 1.8%) events are trapped
- 0 are reflected

Trapped Events: Initial Vertex and Angle

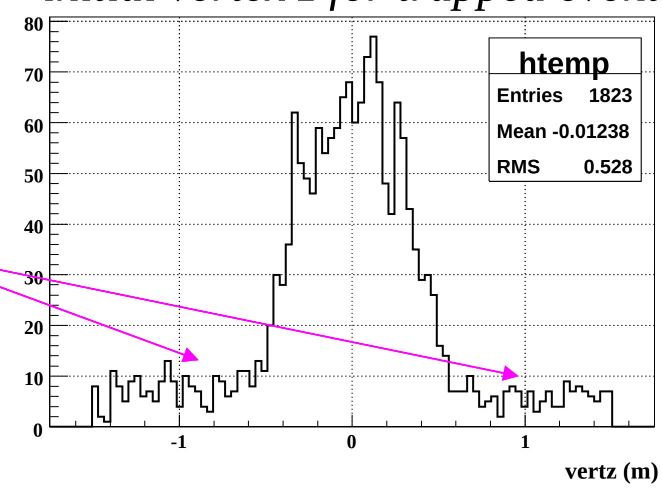
Initial pitch angle vs vertex z, trapped events



initial pitch angle for trapped events

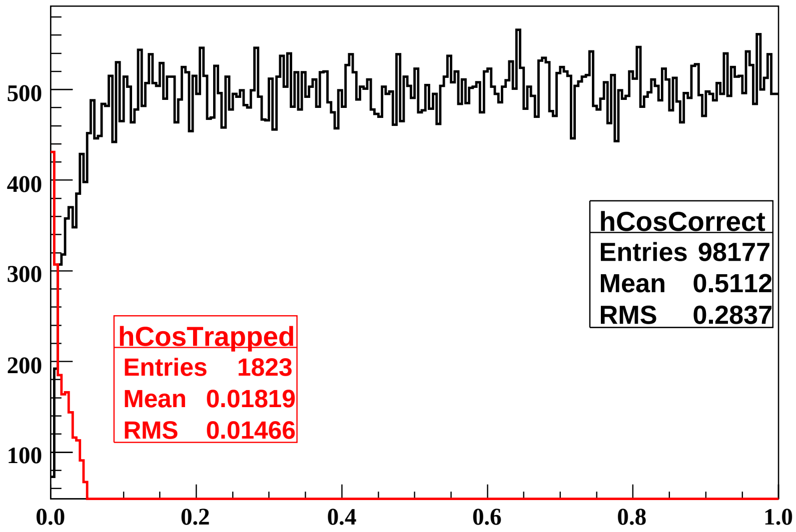


initial vertex z for trapped events



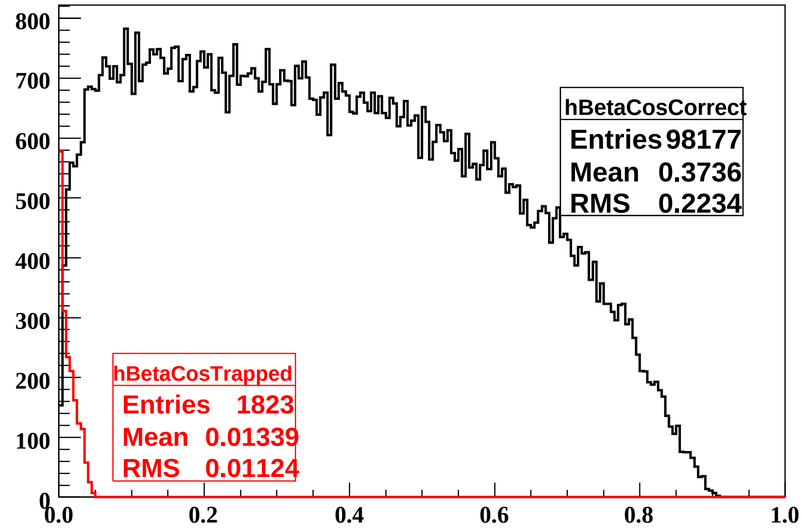
A small fraction of events outside the trap with very large pitch angle can also get trapped

Systematics to Big A I



cosθ distribution for detected and trapped events

βcosθ distribution for detected and trapped events



Systematics to Big A II

$$A_{true} = A\frac{\langle \beta\cos(\theta)\rangle_{\det} N_{\det} + \langle \beta\cos(\theta)\rangle_{trap} N_{trap}}{N_{\det} + N_{trap}}$$

$$= A\frac{98177\times 0.3736 + 1823\times 0.01339}{100000} = 0.3670A$$

Calculate **measured** asymmetry under three scenarios

A. None of the trapped electrons are detected (extreme bias, impossible in reality)

B. All trapped electrons are detected after scattering, with complete loss of memory of initial momentum direction

C. All trapped electrons end up in the WRONG side of the detector after scattering (extreme bias)

Systematics to Big A III

Case A

$$A_{meas} = A\frac{\langle \beta\cos(\theta)\rangle_{\det} N_{\det}}{N_{\det}} = \frac{98177\times 0.3736}{98177}A = 0.3736A$$

Case B

$$A_{meas} = A\frac{\langle \beta\cos(\theta)\rangle_{\det} N_{\det} + 0N_{trap}}{N_{\det}+N_{trap}} = \frac{98177\times 0.3736+0}{100000}A = 0.3668A$$

Case C

$$A_{meas} = A\frac{\langle \beta\cos(\theta)\rangle_{\det} N_{\det} - \langle \beta\cos(\theta)\rangle_{trap} N_{trap}}{N_{\det}+N_{trap}}$$

$$= \frac{98177\times 0.3736 - 1823\times 0.01339}{100000}A = 0.3665A$$

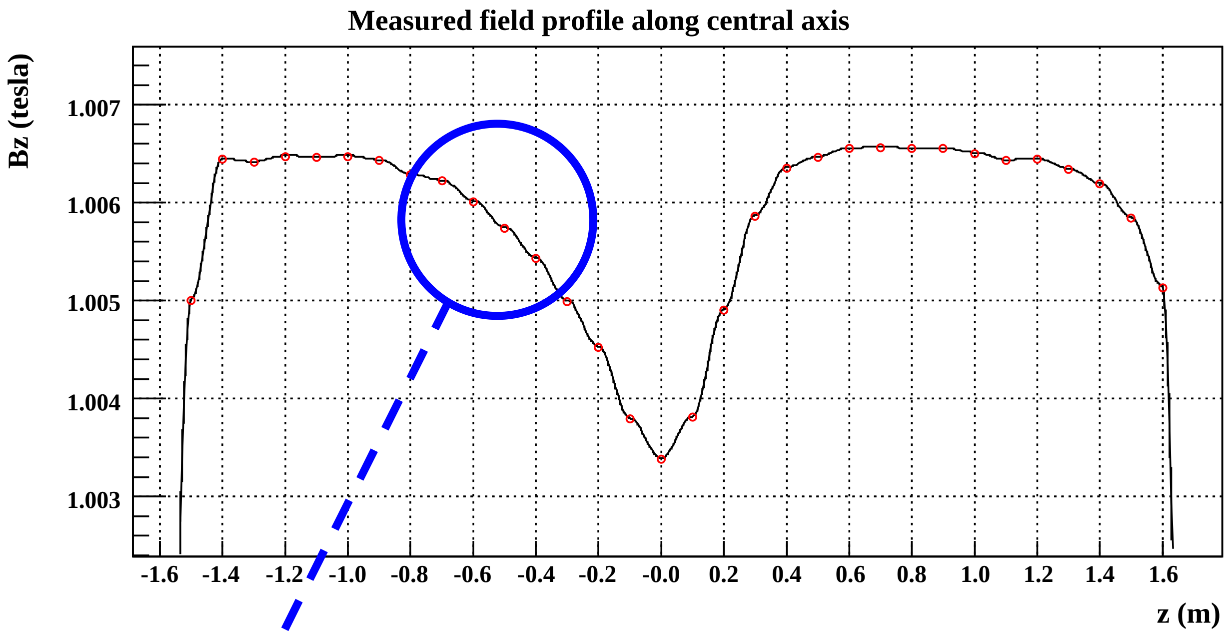
Discussion

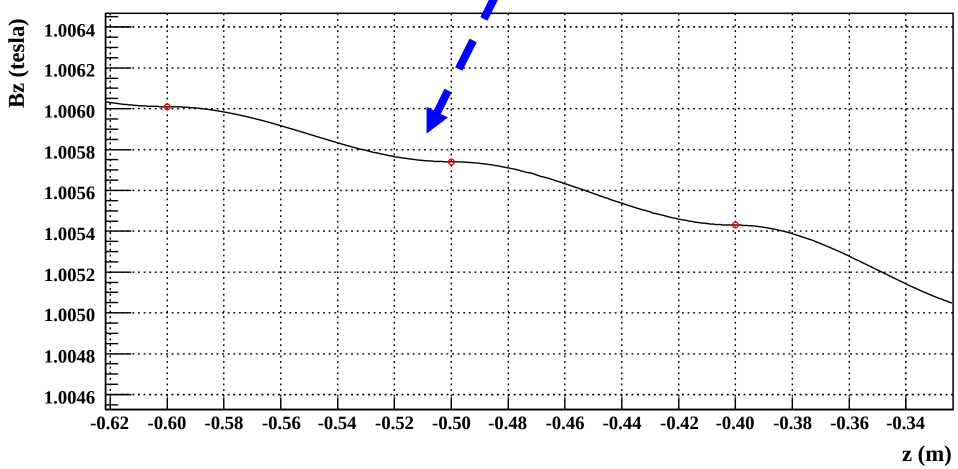
Case	$(A_{true}-A_{meas})/A_{true}$
A	-1.8%
B	0.067%
C	0.13%

- If the "trapped" events were lost permanently (case A), then 30 Gaus dip would produce non-negligible bias to the measured asymmetry due to the selective angular acceptance
- In reality, the "trapped" events will eventually escape and be detected due to the scattering processes. Realistically, the bias to the asymmetry is <0.07% (case B).

Part II: Real field profile, real experiment

Implementing Real Field Configuration

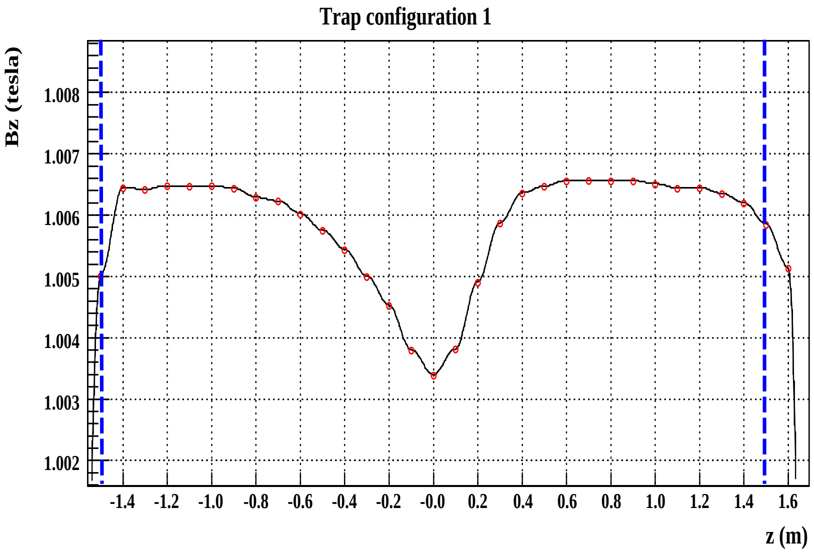


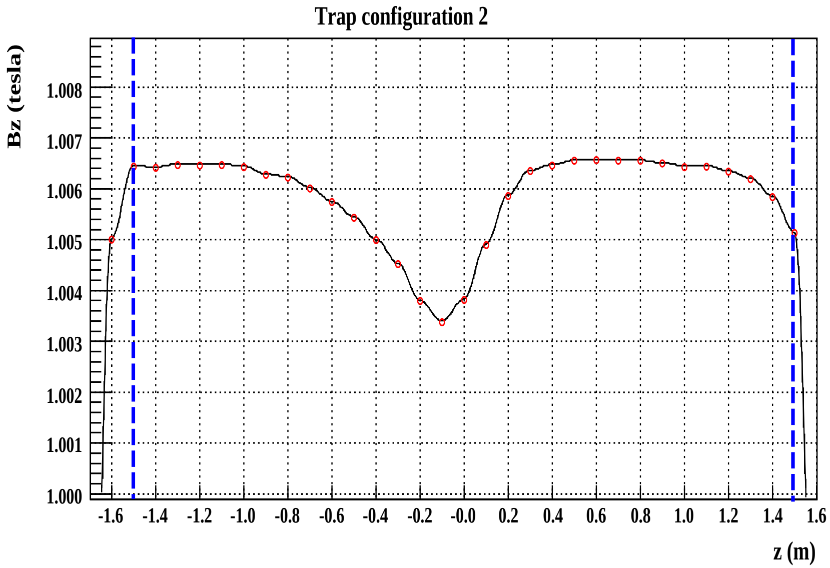


- Use Bz ~ cos(z) function to connect measured points. This will guarantee a smooth function!
- Satisfy $\nabla \cdot B = 0$

Positioning of the Decay trap

- The field was mapped by 32 points (with 10 cm spacing) within 310 cm. We do not know exactly where the decay trap is relative to the map.
- So I assumed two extreme cases as shown below (blue lines show the ends of the decay trap)





Events Sorting

Unlike the ideal field profile, reflections also occur!

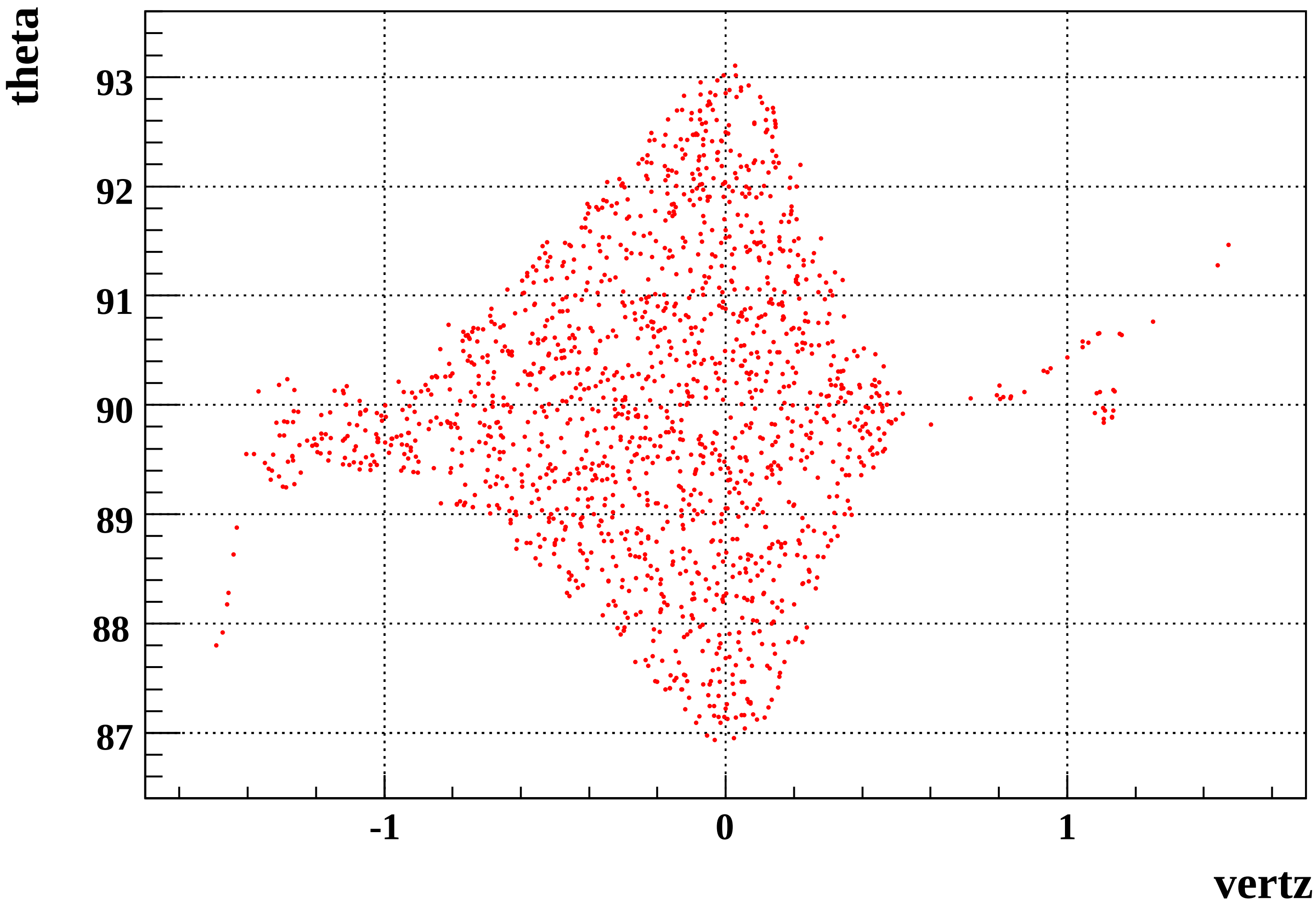
Trap configuration 1:

- Total = 100 K
- Correct = 98221
- Trapped = 1579
- Initially left, reflected toward right = 114
- Initially right, reflected toward left = 86

Trap configuration 2:

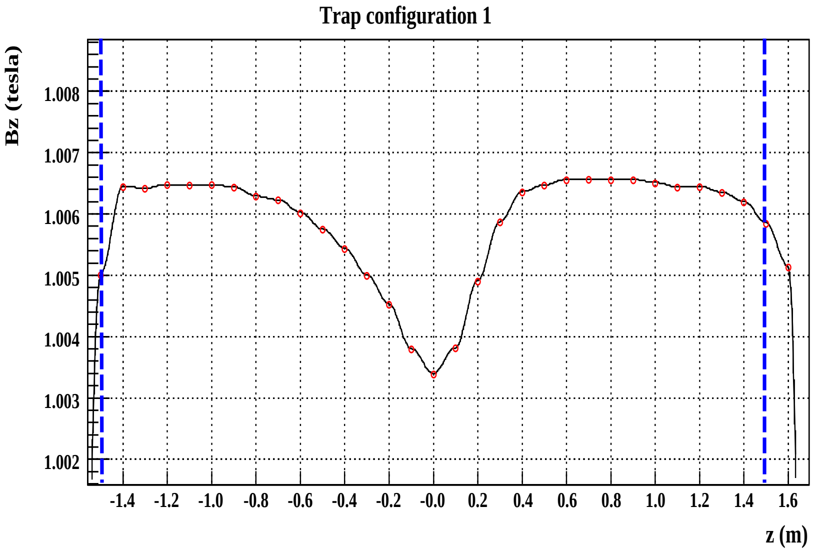
- Total = 100 K
- Correct = 98221
- Trapped = 1580
- Initially left, reflected toward right = 158
- Initially right, reflected toward left = 41

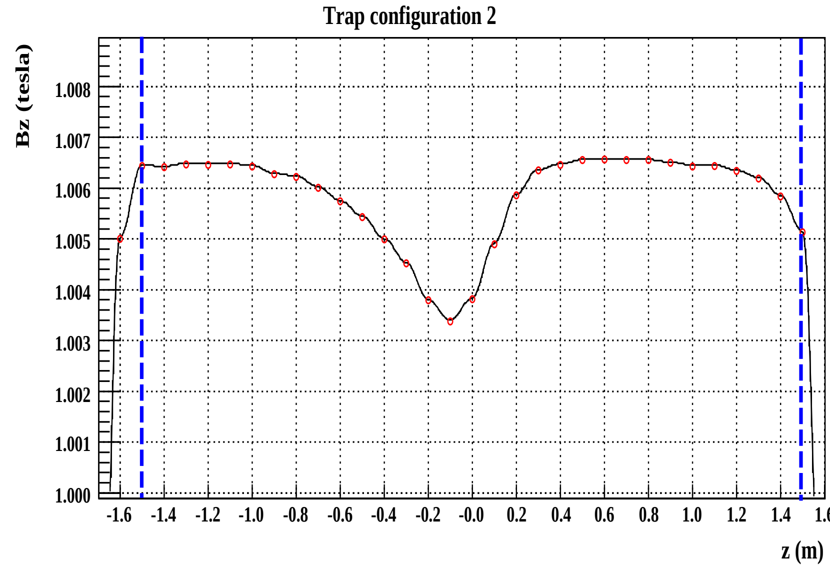
Trapped Events

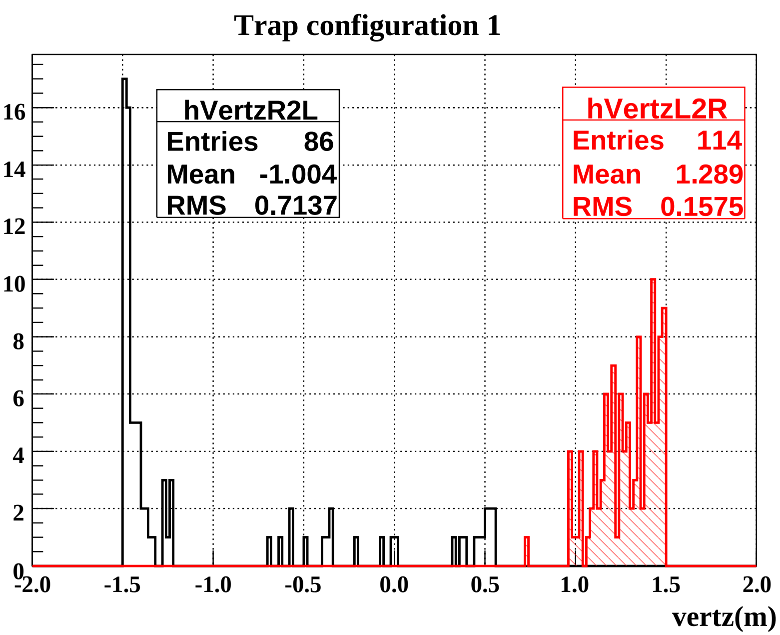


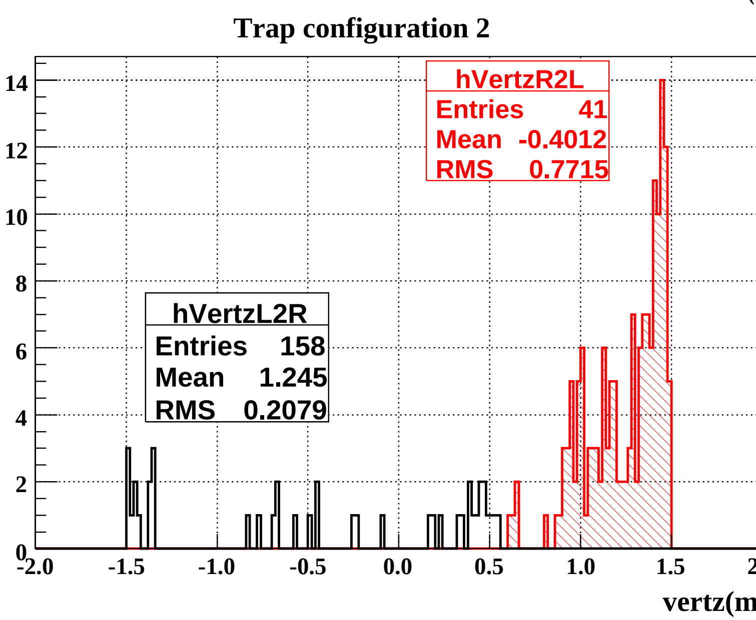
Asymmetric initial vertex distribution for the trapped electrons reflects the “shape” of the trap.

Reflected Events









Asymmetry in reflected events due to asymmetric field at trap ends.

Systematics to Big A

$$A_{true} = A\frac{\langle\beta\cos(\theta)\rangle_{corr} N_{corr} + \langle\beta\cos(\theta)\rangle_{L2R} N_{L2R} + \langle\beta\cos(\theta)\rangle_{R2L} N_{R2L} + \langle\beta\cos(\theta)\rangle_{trap} N_{trap}}{N_{corr} + N_{L2R} + N_{R2L} + N_{trap}}$$

$$A_{meas} = A\frac{\langle\beta\cos(\theta)\rangle_{corr} N_{corr} - \langle\beta\cos(\theta)\rangle_{L2R} N_{L2R} - \langle\beta\cos(\theta)\rangle_{R2L} N_{R2L} - \langle\beta\cos(\theta)\rangle_{trap} N_{trap}}{N_{corr} + N_{L2R} + N_{R2L} + N_{trap}}$$

- Reflected events are just like back-scattered event. They caused the "+" to "–" flip.
- Assumed all trapped events are detected by ended up into the wrong side of the detector → "–" (case C on page 10)

Results

Trap config 1	Events	$\langle\beta\cos\theta\rangle$	Rate fraction (%)	Fractional bias(%)
Correct	98221	0.3716	98.2	
Trapped	1579	0.01306	1.58	-0.1129
Left→Right	114	0.00618	0.11	-0.004
Right→Left	86	0.01175	0.09	-0.006
Total	100000			-0.12

Trap config 2	Events	$\langle\beta\cos\theta\rangle$	Rate fraction (%)	Fractional bias (%)
Correct	98221	0.3715	98.2	
Trapped	1580	0.01306	1.58	-0.1130
Left→Right	158	0.007564	0.16	-0.007
Right→Left	41	0.01067	0.04	-0.002
Total	100000			-0.12

- So reflected events have completely neglible systematics to A
- Trapped events is more likely to be washed out (as opposed to be “reflected” in the table). Then the bias will be ~0.06% level.

“Lifetime” of Electron in the Trap

We have assumed that initially trapped electrons will get out after scattered with residual gas molecules in the spectrometer vacuum (1e-6 Torr).

Q: how long it takes the electrons to get out the trap?

Since initially only large pitch angle electrons (>86.87 deg) are trapped, it only takes **a few degrees of angular deflection** in the electron-molecule scattering to “untrap” those electrons.

Procedure:

- Use a hand-calculation to estimate the order of magnitude
- A GEANT4 simulation with varying vacuum level (down to 1e-3 Torr level) and extrapolate the results to 1e-6 Torr vacuum

Simple Estimate

- Only consider single scattering
- Use Mott cross-section
- Integrate Mott cross section from θ_0 to π, where θ_0 is a few deg

$$\frac{d\sigma_{Mott}}{d\Omega}(\theta) = \frac{Z^2\alpha^2}{4|\vec{p}|^2\beta^2\sin^4(\frac{\theta}{2})}(1-\beta^2\sin^2(\frac{\theta}{2}))$$

$$\sigma_{\theta_0} \equiv \int_{\theta_0}^{\pi}\frac{d\sigma_{Mott}}{d\Omega}d\Omega$$

- Calculate mean free path and “trap time”
  - A is atomic number of the molecule
  - ρ is gas density (linear in vacuum level)
  - N_A is the Avogadro’s number

$$\lambda = \frac{A}{\rho\sigma_{\theta_0}N_A}, \tau = \frac{\lambda}{\beta c}$$

Also see

http://cambot.caltech.edu/ucna/bckscat_simple_calc_v1.pdf

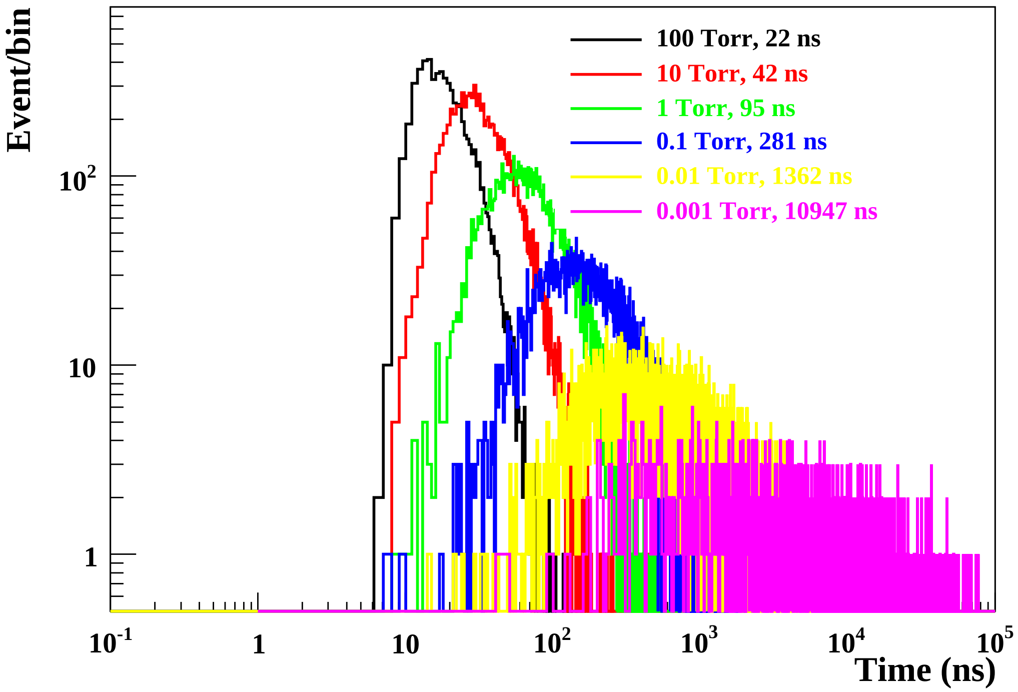
Results

Assume a N_2 gas of 1e-5 Torr

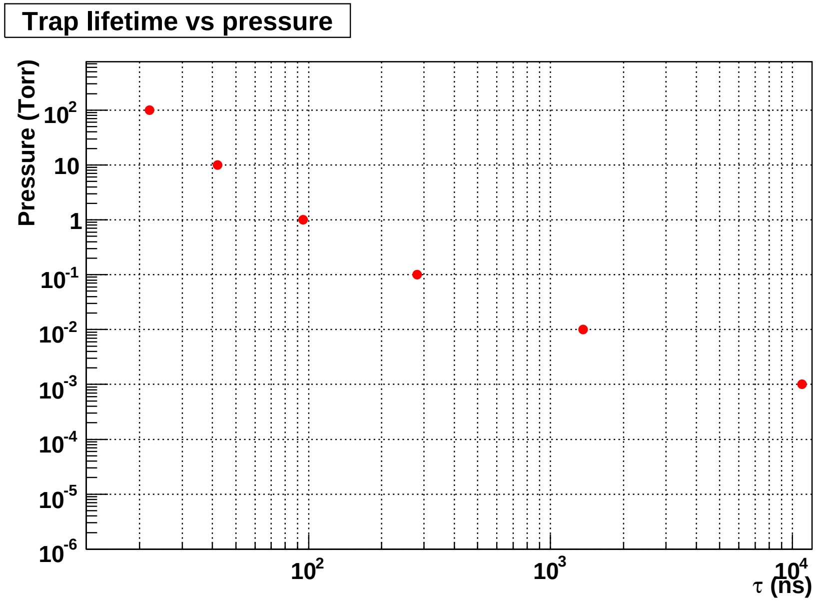
E_e (keV)	θ_0 (deg)	λ (km)	τ (ms)
100	2	30	0.2
100	5	240	1.5
100	10	1061	6.5
300	2	210	0.9
300	5	1706	7.3
300	10	7602	33
500	2	497	1.9
500	5	4049	16
500	10	18103	70

Simulation of the Trap Lifetime

- Use real field profile
- Varied N2 gas pressure: 100, 10, 1, 0.1, 0.01, 0.001 Torr
- **Generate neutron betas with large pitch angle that satisfy trapping condition from the center of dip**
- Record the time the electrons escape the decay trap (±1.5 m)



Trap Lifetime, Extrapolated



The pressure scaling works better for lower pressure, indicating that the single scattering assumptions works for pressure below 1×10^{-3} Torr.

Therefore, at 1×10^{-5} Torr, the extrapolated trap lifetime is $\sim10^{6}$ ns ~ ms. This is in agreement with the hand estimation.

Conclusion

- The field dip in SCS will trap ~1.5% of the neutron betas
- Under experimental vacuum (1×10^{-5} Torr), these electrons escape from the trap at ~mili-second time scale with momentum direction randomized
- Due to large pitch angle of these initially trapped electrons, the fractional bias to the measured asymmetry is at a negligible 0.06% level.